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REVERSE DERIVATIONS ON SEMI PRIME RINGS

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ABSTRACT

In this paper, we prove some results concerning to reverse derivations on semi primerings are presented. We prove that let d be a commuting reversed erivation of a semi primering R. Then $\alpha \in \Delta$ (d) if and only if $\alpha \in Z$ and ad(x2)= 0, for all $x \in R$

KEYWORDS: Derivation, Reverse Derivation, Semi Prime Ring, Center

INTRODUCTION

I.N.Herstein [4]has introduced the concept of reverse derivations of prime rings and proved that anon-zero reversed erivation$*$ of a primering Aisa commutative integral domain and$*$ is an ordinary derivation of A. Later Bresar and Vukman [2] have studied the notion of reversed erivation and some properties of reversed erivations. M. Samman and N. Alyamani [6] have studied some properties of reversed erivations on semi primering s and proved that amapping d on as emiprimering R is are verse derivation if and only if, it is a central derivation. Also proved that if a primering R admitsa non-zero reverse derivation, then R is commutative. K. Suvarna and D.S. Irfana [7] studied some properties of derivation so n semi primerings. Laradji and Thaheem [5] first studied the dependent elements in endomorphisms of semiprimerings and generalized a number of results of [3] for semiprimerings. Ali and Chaudhry [1] investigated the decomposition of as emiprimering R using dependent elements of a commuting derivation d.

Preliminaries

An additive map d fromaring R to R is called a derivation if $d(xy)=d(x)y+xd(y)$for all x, yi n R.An additive map d from a ring R to R is called are verse derivation if $d(xy)=d(y)x+yd(x)$for all x, y in R.A mapping d:$R\to R$ is called commuting derivation if$[d(x),x]=0$, for all x in R.A mapping d:$R\to R$is called commuting reverse derivation if [x, d(x)] = 0, for all x in R. A ring R is called semi prime if x ax = 0 implies x = 0 for all x, a in R. Through out this paper$R$will denote as emiprime ring, $D(d)$ is the collection of all dependent elements of d and Z its center.

MAIN RESULTS

Theorem1: If d is a commuting in nerreverse derivation on as emiprimering R, then $d=0$. Now, we prove the following result:

Theorem2: Let $d$bea commuting reversed erivation of a semiprimering R.Then$a \in D(d)$if and only if $a\in Z$ and $ad(x^2)=0$, for all $x\in R$.

Proof: Let $a\in D(d)$. Then,

$ad(x) = a[x, a]$, for all $x \in R$ (1)

If we replace x by $y\,x$ in equ.(1), then we get,

$\Rightarrow ad\ (yx) = a[yx, a]$

$\Rightarrow a\ (d(y)\ x + yd(x)) = a(y[x, a] + [y, a]x)$

$\Rightarrow ad(y)\ x + a\,yd(x) = ay[x, a] + a[y, a]x$, for all $x, y \in R$ (2)

From equ.'s (1) and (2), we get,

$\Rightarrow a[y, a]\ x + ayd(x) = ay[x, a] + a[y, a]x$

$\Rightarrow ay\,d(x) = ay[x, a]$, for all $x, y \in R$ (3)

If we multiply equ. (3) by z on the left, then we get,

$\Rightarrow z\ ayd(x) = zay[x, a]$ (4)

By replacing y by $z\ y$ in equ. (3), we get,

$\Rightarrow azyd\ (x) = azy\ [x, a]$ (5)

By subtracting equ. (5) From equ. (4), we get,

$\Rightarrow zay(x) - azy\,d(x) = zay\ [x, a] - azy\ [x, a]$

$\Rightarrow (\ za - az)yd(x) = (za - az)y[x, a]$

$\Rightarrow [z, a]\,yd(x) = [z, a]\,y[x, a]$ (6)

By multiplying equ. (6) by x on the right, we get,

$\Rightarrow [z, a]\,yd(x)x = [z, a]\,y[x, a]x$ (7)

If we replace y by yx in equ. (6), then we get,

$\Rightarrow [z, a]\ yxd(x) = [z, a]\,yx\ [x, a]$ (8)

By subtracting equ. (7) from equ. (8), then we get,

$\Rightarrow [z, a]\,yxd(x) - [z, a]\,yd(x)x = [z, a]\,yx[x, a] - [z, a]\,y[x, a]x$

$\Rightarrow [z, a]\,y(xd\ (x) - d(x)x) = [z, a]\,y(x[x, a] - [x, a]x)$

$\Rightarrow [z, a]\,y[x, d(x)] = [z, a]\,y[x,[x, a]]$ (9)

Since d is commuting, from equ.(9), we get,

$\Rightarrow [z, a]\,y[x,[x, a]] = 0$ (10)

If we multiply equ. (10) by z on the left, then we get,

$\Rightarrow z[z, a]y[x,[x, a]]=0$ (11)

Now we replace y by zy in (10), then we get,

$\Rightarrow [z, a]zy[x,[x, a]]=0$ (12)

By sub tracting equ.(12)from equ.(11), then

$\Rightarrow z[z, a]y[x,[x, a]]-[z, a]\ zy\ [x,[x, a]]=0$

$\Rightarrow (z[z, a]-[z, a]z)y[x,[x, a]]=0$

$\Rightarrow [z,[z, a]]y[x,[x, a]]=0$

Replace z by x in the above equation, then we get,

$\Rightarrow [x,[x, a]]y[x,[x, a]]=0$

By using the semi primenes s of R, we get,

$\Rightarrow [x, [x, a]]=0$, for all $x\in R$ (13)

Thus the inner derivation $\psi : R\rightarrow R$ defined by $\psi(x)=[x, a]$ is commuting.

Hence $\psi(x)=0$ byTheorem:1,whichimplies $[x, a]=0$ .Thus $a\in Z$.Further from

Equ .(1),we get, $ad(x)=0$.

Now $ad(x^2)=a(d(x)x+xd(x))$

$=ad(x)\ x+axd\ (x)$

$=axd(x)$, since $a\in Z$

$=xad(x)$

Therefore, $ad(x^2)=0$

Conversely, let $a\in Z$ and $ad(x^2)\ =0$. Then, $ad(x^2)\ =0$ implies $ad(x)\ x+axd\ (x)=0$.

Since d is commuting, $ad(x)\ x+ad\ (x)x=0$

$\Rightarrow 2ad(x)\ x=0$

Since R is of char $\neq 2$, $ad(x)x=0$.

By multiplying the above equation by $ad(x)$ on the right, we get,

$Ad\ (x)xad(x)=0$.

Since R is semi prime, then $ad(x)=0=a[x, a]$.

Hence $a \in D(d)$

This completes the proof of the theorem.

Corollary1: Let R be a semi primer in g and d a commuting reverse derivation of R. Let $a \in D(d)$, then $d(a)=0$.

Proof: Since $a \in D(d)$, then

$ad(x^2)=0$ implies $ad(x)=0$, for all $x \in R$ (14)

We replace x by $d(x)$ in equ. (14), then

$\Rightarrow ad(d(x))=0$

$\Rightarrow ad^2(x)=0$, for all $x \in R$ (15)

From equ.(14), we get,

$\Rightarrow d(ad(x))=0$

$\Rightarrow d(0)=0$, which implies that,

$\Rightarrow d(x)\ d(a)+ad^2(x)=0$

By using equ.(15),we get,

$\Rightarrow d(x)\ d(a)=0$ (16)

We replace x by $x\,a$ in equ. (16) And using equ. (16) again, we get,

$\Rightarrow d(xa)\ d(a) = 0$

$\Rightarrow (d(a)\ x + ad(x))\ d(a) = 0$

$\Rightarrow d(a)\ xd(a) + ad(x)d(a)=0$

$\Rightarrow d(a)\ xd(a)=0$, for all $x \in R$

By using the semi primenes s of R, we get, $d(a)=0$.

Corollary2: Let R be as emiprimering and da commuting reversed erivation of R. Then $D(d)$is a commutative semi prime subring of R.

Proof: Let a, $b \in D(d)$.Then by Theorem: 2, a, b$\in Z$, $ad(x)=0$ and $bd(x)=0$, For all $x \in R$. Obviously $a-b \in Z$ and $abd(x)=0$.So,$a-b$ and $ab \in D(d)$.

Since $a,b \in Z$,so,$ab=ba$ .Thus $D(d)$ is a commutative subring of R. To show semiprimeness of $D(d)$, we consider $aD(d)=0$, $a \in D(d)$. Then $axa=0$, for all $x \in D(d)$.

In particular $a^3=0$, which implies $a=0$ because R has no central nil potents. Thus $D(d)$ is a commutative semi prime subring of ring R.

Corollary3: Let R be a commutatives emiprimering and d are verse derivation of R. Then $D(d)$is an ideal of$R$.

Proof: Since R is commutative, so, d is commuting. Let $a,\ b \in D(d)$ .Then by Corollary:2,$a-b \in D(d)$ .Let $a \in D(d)$ and $r \in R$. Then $ad(x)=0$, for all $x \in R$ .Thus$rad(x)=0$ .Since$ar=ra$,so$rad(x)=ard(x)=0$,forall$x \in R$ .Hence $ar=ra \in D(d)$. Thus $D(d)$ is an ideal of R.

Remark 1: If R is as emiprimering and U an ideal of R, then it Is easy to verify that U is as emi prime subring of R and $Z(U) \subseteq Z$. ■

Remark 2: If d is a commuting reverse derivation on R and $a \in D(d)$. Then by Theorem:2,

$Ad\ (x)\ =0$,forall$x \in R$.Thisimplies$0=ad(xy)=ad(y)x+a\,yd(x)=ayd(x)$,

Which gives $ayd(x)=0$ .Thus$d(x)ayd(x)a=0$,forall$x,y \in R$, which by

Semi primeness of Rimplies $d(x)\ a=0$

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